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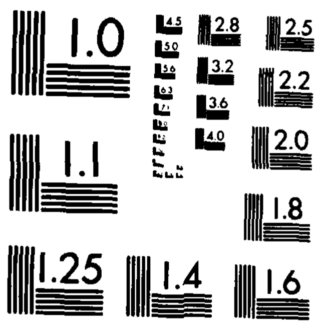

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A NOTE ON "GEOMETRIC TRANSFORMS" OF DIGITAL SETS

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ABSTRACT

Document defines ~~We define~~ a "geometric transform" on the digital plane as a function f that takes pairs (P,S), where S is a set and P a point of S, into nonnegative integers, and where f(S,P) depends only on the positions of the points of S relative to P. Transforms of this type are useful for segmenting and describing S. Two examples are "distance transforms," for which f(S,P) is the distance from P to $\bar{S}$, and "isovist transforms," where f(S,P) is (e.g.) the area of the part of S visible from P. This note characterizes geometric transforms that have certain simple set-theoretic properties, e.g., such that $f(S\cap T, P)=f(S,P)\wedge f(T,P)$ for all S,T,P. It is shown that a geometric transform has this intersection property if and only if it is defined in a special way in terms of a "neighborhood base"; the class of such "neighborhood transforms" is a generalization of the class of distance transforms.

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1. Introduction

Given a subset S of a digital picture, there are various useful ways of defining functions on S that associate with each point P of S some geometric property of S "relative to P". An early example [1] is the distance transform, which associates with each $P \in S$ the distance (with respect to some given metric) from P to $\overline{S}$ (the complement of S). This transform is a useful tool for describing or segmenting S; for example, the well-known "medial axis transformation" of S is just the set of local maxima of its distance transform. A more recent example [2] is the class of "isovist transforms", which associate with each P some property of the part of S "visible" from P, e.g., its area; such transforms can be used, e.g., to find minimal sets of points from which all of S can be seen. (A point Q of S is said to be visible from P if the straight line segment $\overline{PQ}$ lies entirely in S.)

In this note we give a general definition of such "geometric transforms" (for brevity: G-transforms). We also characterize G-transforms that have certain simple properties with respect to set-theoretic operations. In particular, we consider G-transforms having the "intersection property": for any two sets S and T, the transform values for $S \cap T$ are (pointwise) the infs of the values for S and for T. We show that a G-transform has this property iff it can be defined in a special way in terms of a "neighborhood basis"; the class of such transforms includes the class of distance transforms. Interestingly, the analogously defined "union property" implies that the transform must be trivial.

2. G-transforms

Let Σ be a bounded set of lattice points in the plane (e.g., a digital picture), let 2^{Σ} be the set of subsets of Σ, and let f be a function defined on $2^{\Sigma}\times\Sigma$. For simplicity, we shall assume that f is integer-valued; that $f(S,P)=0$ whenever $P\notin S$; and that $f(S,P)>0$ whenever $P\in S$. We call f a G-transform if $f(S,P)$ depends only on the positions of the (other) points of S relative to P. This is a rather general definition; the following are a few examples of G-transforms:

a) The characteristic function, i.e., $f(S,P)=1$ iff $P\in S$

b) The distance transform, i.e., $f(S,P)=$the distance from P to $\overline{S}$

c) The "area transform": $f(S,P)=$the area of the connected component of S that contains P

d) The isovist transform: $f(S,P)=$the area of the part of S visible from P

Since a G-transform is defined in terms of positions relative to P, it is evidently shift-invariant -- in other words, shifting S cannot change the G-transform values of its points.* In particular, we have

Proposition 1. $f(\{P\},P)$ has the same value for any P. ||

For simplicity, we assume that this value is 1.

*We assume that when S shifts, it remains inside Σ. Alternatively, we could allow cyclic shifts, and define $f(S,P)$ in terms of the positions of the points of S relative to P "modulo Σ".

We say that f has the union property if $f(S \cup T,P)=f(S,P) \vee f(T,P)$ for all S,T,P, and the intersection property if $f(S \cap T,P)=f(S,P) \wedge f(T,P)$ for all S,T,P. Evidently the characteristic function has both the union and the intersection property. In fact, it is the only G-transform that has the union property, as we see from

Proposition 2. A G-transform f has the union property iff it is the characteristic function.

Proof: By Proposition 1, $f(\{P\},P)=1$ for all P. It follows from the union property that $f(\{P,Q\},p)=f(\{P\},P) \vee f(\{Q\},P)=1$ for all $\{P,Q\}$, i.e., for any two-element subset of Σ. By induction, the same is true for any finite subset of Σ. ‖

The G-transforms that have the intersection property are less trivial; we shall characterize them in the next section.

3. N-transforms

Let n: $\{O\}=N_0 \subsetneq N_1 \subsetneq N_2 \subsetneq \ldots$ be a nested set of finite subsets of Σ that contain the origin O. For any point P, let N_{Pi} be the result of shifting N_i to bring O into the position of P; thus n_P: $\{P\}=N_{P0} \subsetneq N_{P1} \subsetneq N_{P2} \subsetneq \ldots$ is a nested set of sets that contain P. We call n_P a <u>neighborhood basis</u> for P.

Let $1=n_0 \le n_1 \le n_2 \le \ldots$ be any monotonic nondecreasing sequence of positive integers. For any $S \in 2^{\Sigma}$ and any $P \in S$, there is a largest i, call it i(S,P), such that $N_{Pi} \subseteq S$. (Note that $N_{P0}=\{P\} \subseteq S$, and that S is finite.) Let the G-transform f be defined by $f(S,P)=n_{i(S,P)}$. We call such a G-transform an <u>N-transform</u>.

It is easily verified that a distance transform is a N-transform. In fact, let N_i be the "disk" of radius i centered at O, i.e., the set of points whose distances from O are $\le i$, and let $n_i=i+1$; then the distance transform f(S,P) is just n_{Pi} (1 greater than the radius of the largest disk centered at P and contained in S). Note also that the characteristic function is an N-transform, if we simply take $n_i=1$ for all i.

<u>Theorem 3</u>. A G-transform f has the intersection property iff it is an N-transform.

<u>Proof</u>: For any S and T we have $i(S\cap T,P)=i(S,P)\wedge i(T,P)$, since the N_P's are nested. Thus if f is an N-transform we have $f(S\cap T,P)= n_{i(S,P)\wedge i(T,P)}=n_{i(S,P)}\wedge n_{i(T,P)}$ (since the n's are monotonic)$=$ $f(S,P)\wedge f(T,P)$, so that f has the intersection property.

Conversely, let f be a G-transform and have the intersection property. For any k, if $f(S,P)=f(T,P)=k$, we have $f(S\cap T, P)=k$; thus if there are any sets S such that $f(S,P)=k$, there is a smallest such set, call it S_{Pk}. By shift invariance, $f(S,P)=k$ implies $f(S',P')=k$, where S' is S shifted to make P coincide with P'; thus $S_{P'k}$ exists iff S_{Pk} does, and they are translates of one another. Let $1=k_0<k_1<\ldots$ be those k's for which S_{Pk} exists; then $n_P: \{P\}=N_{P0}\subsetneqq N_{P1}\subsetneqq\ldots$, where $N_{Pi}=S_{Pk_i}$, is a neighborhood basis for P. Moreover, for any S, let $i(S,P)$ be the largest i such that $N_{Pi}\subseteq S$, and let $f(S,P)=m$. If we had $m=k_j>k_i$, S would have to contain $S_{Pk_j}=N_{Pj}$, contradicting the definition of i. On the other hand, if $m=k_h<k_i$, by the intersection property $k_i=f(N_{Pi},P)=f(S\cap N_{Pi},P)=f(S,P)\wedge f(N_{Pi},P)=k_h$, contradiction. Hence $f(S,P)=k_i$, so that f is an N-transform. ‖

Thus we see that the intersection property characterizes a class of G-transforms that constitute a natural generalization of the distance transforms.

4. Concluding remarks

The main result of this note has been a "set-theoretic" characterization of the "distance-like" G-transforms. It would be of interest to develop characterizations of other useful classes of G-transforms.

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2. L. S. Davis and M. L. Benedikt, Computational models of space: Isovists and isovist fields, Computer Graphics Image Processing 11, 1979, 49-72.

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